Assessing bridge stiffness decline using power spectral density: A case study on Saigon Bridge

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Abstract:

Bridges, especially civil infrastructure, serve a vital role in social activities. A damaged bridge can cause severe consequences such as traffic congestion, economic impacts, and loss of lives. Therefore, it is crucial for infrastructure management communities to detect and assess the condition of damaged bridges to ensure safety. To address this issue, in this study, the authors have put forth a novel set of parameters that can be used to evaluate the reduction in stiffness of spans over time. The stiffness is a critical factor that determines its structural integrity, and it is essential to monitor any changes in this parameter over the lifespan of the bridge. The proposed parameters take into account factors such as the age of the bridge, the material used in its construction, and environmental factors that may contribute to the degradation of the structure. They conducted experiments on a beam structure to evaluate its ability to monitor the reduction in bearing capacity of Saigon Bridge's spans using these parameters. To analyze the vibration signals from a randomized traffic load model, they utilized the power spectral density. The results of the study suggest a correlation between the frequency of harmonics and the high-frequency regions observed in the power spectral density and the decline in the stiffness of spans.

Keywords: Bridge; Civil infrastructure; Damage detection; Condition assessment; Stiffness decline; Power spectral density; Saigon Bridge; Bearing capacity reduction.

1. Introduction

In some cases, a defect in a structure may not cause changes in all natural frequencies. When a natural frequency remains unchanged, it cannot be used as an indicator of damage, but it can still help identify the location of the crack [1]. This may indicate that the crack commonly occurs at a certain nodal point opposite to the structure's shape, and random loading conditions lead to inaccurate measurements when determining the frequency of the structure, making it difficult to detect frequency changes in reality. However, some cases have shown that damage can result in an increase in natural frequencies. For example, in pre-stressed concrete beams, a decrease in pre-stresses can lead to an increase in the modulus of elasticity of concrete, resulting in an increase in natural frequencies.

Various [2]–[3] studies have investigated the use of frequency change methods to detect defects, which involve measuring the frequency shifts resulting from known types of damage. Vandiver conducted a study on an offshore lighthouse to detect damage by evaluating the frequency changes in the first two bending and twisting modes. Numerical analysis shows that actual mass changes of the tower caused by liquid sliding in fixed tanks on the platform will result in frequency changes of no more than 1% of the natural frequency value. This indicates that such changes are also insignificant for the natural frequencies of all three states considered in this study, including bending, torsion, and bending-torsion combined. However, the study demonstrated that most component failures would result in frequency changes greater than 1%, making it possible to detect damage in most components.

Kenley and Dodds [4] conducted a study on changes in resonant frequencies caused by cracks in an offshore drilling rig that had been inactive for a long time. The study aimed to examine the impact of natural frequencies on the service life of the structure through the influence of environmental conditions. The study found that changes in resonant frequency values do not reflect changes in the structure when it cannot detect the complete failure of a diagonal, horizontal, or cross member of the space frame. The study can only detect damage when the structure decreases 5% in overall stiffness and only changes no more than 1% in resonant frequency values for different structural behavior states [5]-[6]. Although there are many other methods proposed to evaluate structural changes, such as using the ratio of natural frequency changes for different oscillation modes and combining them in the Stubbs process [7], the study also combines with the theory of phase transformation to shows the internal state of the structure to detect damage in composite materials. However, their effectiveness is often low due to small frequency changes or changes in the shape of oscillation modes that make them not suitable for reality [8]-[9]. A new approach involves using a crack model in a simplified evaluation process to estimate the location and size of cracks by the difference between the measured natural frequency values and predicting these changes through updates to artificial intelligence models or prediction algorithms [10]-[12]. Moreover, this predictive model will simplify input data and be trained for pattern recognition methods to be applicable in monitoring the health of actual structures [13].

This article focuses on assessing the load-bearing capacity of cable-stayed bridges, a simple structure that is suitable for practical conditions in Vietnam. The research model will focus on the Saigon Bridge through the analysis of random oscillation signals on the span [14]. This is a particular challenge in this study because the materials and structures of the cable-stayed bridges are different within the same bridge. Based on previous studies, the authors used vibration measurement methods to evaluate the changes in the bridge structure. Specifically, the study attempted to evaluate the changes in the structure of cable-stayed bridges by using the shape change of power spectral density obtained from analyzing the actual vibration signals of the model under random traffic loads. This study shows the presence of frequency harmonics and areas containing high-frequency values in the power spectral density, which are related to the decreasing stiffness of the cable-stayed bridges over time. The results of this study indicate that the changes in frequency harmonics and the shape of the spectrum are conditions for evaluating and maintaining the structure. This is also the basis for monitoring the condition of the cable-stayed bridge structure to ensure safety and durability.

2. Theory background

2.1. Fourier transform and frequency spectrum

The combination of sine and cosine functions, known as the Fourier series, can be utilized to represent most functions. These series are made up of coefficients that provide information about the amplitudes of each sine and cosine function with their respective frequencies. However, when dealing with practical measurement situations, it may not be possible to know the input signal in its functional form. As a result, Fourier analysis theory, which suggests that any function can be expressed as a Fourier series, has not been widely used as a specific technique for analyzing measured signals. To address technical issues, many studies have proposed a wave analysis technique that has been developed in recent years to separate measured dynamic signals into separate components, including their frequency and amplitude. Equation (1) describes the dynamics of the measured signal performed over any cycle.

$$A_n = \frac{1}{T}\int_0^{\frac{T}{2}} x(t)\cos n(2\pi f)t.dt \quad ; B_n = \frac{1}{T}\int_0^{\frac{T}{2}} x(t)\sin n(2\pi f)t.dt \tag{1}$$

The first equation (1) provides a representation of the dynamic part of a signal for any period, with A_n and B_n representing the amplitudes of the nth frequency in the Fourier series. However, when the period of the function approaches infinity, the constraint of periodicity is no longer applicable. In this case, the Fourier series is transformed into an integral, and the frequency components become infinitely small. Consequently, the coefficients A_n and B_n become continuous functions of frequency, which can be expressed as *A(f)* and *B(f)*, respectively:

$$A(f) = 2\int_0^{\infty} x(t)\cos(2\pi f)t.dt \quad ; B(f) = 2\int_0^{\infty} x(t)\sin(2\pi f)t.dt \tag{2}$$

The Fourier transform is a mathematical method used to decompose the original signal into different individual signal components characterized by their own frequency and amplitude. This allows the signal components to be represented in the frequency domain using coefficients in the Fourier transform, such as A(f) and B(f), which describe the amplitude of different single frequencies of the signal components. By analyzing the signal in this way, the frequency components of the individual signals can be better determined in terms of their properties and characteristics. It is thanks to this that the Fourier transform is widely used in many fields and has many applications in various other fields:

$$Y(f) = A(f) - iB(f) \tag{3}$$

Where $i = \sqrt{-1}$. Then, it can be directly deduced from equation (3) that:

$$Y(f) = \int_{-\infty}^{\infty} x(t)\left(\cos(2\pi f)t - i\sin(2\pi f)t\right).dt \tag{4}$$

By introducing the identity:

$$e^{-i\theta} = \cos\theta - i\sin\theta$$

This results in:

$$Y(\omega) = \int_{-\infty}^{\infty} y(t)e^{-i\omega t}.dt \tag{5}$$

Equation (5) can be reformulated as:

$$Y(f) = \int_{-\infty}^{\infty} y(t)e^{-i2\pi ft}.dt \tag{6}$$

The equations (5) and (6) define the two-sided Fourier transform of *x(t)*. The Fourier transform is a powerful tool that allows us to represent a signal as a continuous function of frequency, denoted by *Y(f)*. By computing the Fourier transform of *x(t)*, we can reveal the amplitude-frequency characteristics of the signal, which may not be apparent in its time-domain representation. On the other hand, if we have knowledge of or measure *Y(f)*, we can retrieve the original signal x(t) by performing the inverse Fourier transform.

$$Y(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} Y(f)e^{-i2\pi ft}.df \tag{7}$$

Equation (7) demonstrates how the inverse Fourier transform can be used to reconstruct the original signal *x(t)* from its amplitude-frequency attributes. The Fourier transform represents the signal as a continuous function of frequency, with magnitude and phase components. By computing the Fourier transform of *x(t)*, we can identify amplitude-frequency characteristics that may not be evident in its time-domain representation. Conversely, if we have knowledge of the Fourier transform *Y(f)*, we can use the inverse Fourier transform to retrieve the original signal *x(t)*.

$$Y(f) = |Y(f)|e^{i\phi(f)} = A(f) - iB(f) \tag{8}$$

To calculate the magnitude of Y(f), the draft calculates the modulus of the complex number Y(f) by taking the square root of the sum of squares of its components, including the real and imaginary components, as shown in (9).

$$|Y(f)| = \sqrt{\mathrm{Re}\left[Y(f)\right]^2 + \mathrm{Im}\left[Y(f)\right]^2} \tag{9}$$

To determine the phase of *Y(f)*, we use the complex argument function, which gives the angle between the real axis and the complex number *Y(f)*. This angle is given by:

$$\phi(f) = \tan^{-1}\frac{\mathrm{Im}\left[Y(f)\right]}{\mathrm{Re}\left[Y(f)\right]} \qquad (10)$$

As mentioned earlier, the Fourier coefficients allow the representation of *x(t)*'s amplitude spectrum in terms of cosine and sine components.

$$C(f) = \sqrt{A(f)^2 + B(f)^2} \qquad (11)$$

And its phase shift can be expressed by its phase spectrum

$$\phi(f) = \tan^{-1}\frac{B(f)}{A(f)} \qquad (12)$$

2.2. The theory of power spectrum density

Analyzing the frequency components of naturally occurring random processes presents a challenge due to the non-periodic nature of the time history x(t) of the sample function. As a result, a discrete Fourier series cannot be used to represent it. Moreover, for stationary processes, the sample function x(t) extends infinitely, making conventional Fourier analysis theory unsuitable. However, this issue can be addressed by studying the autocorrelation function of the process instead of the sample functions themselves:

$$\int_{-\infty}^{\infty} \left|x(t)\right| dt < \infty \qquad (13)$$

In which: $R_x(\tau \to \infty) = 0$ (14)

And the condition: $\int_{-\infty}^{\infty} \left|R_x(\tau)\right| d\tau < \infty$ (15)

The inverse Fourier transform, as well as the Fourier transform of $R_x(\tau)$, are provided:

$$S_x(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} R_x(\tau).e^{-i\omega\tau}.d\tau \qquad (16)$$

And: $R_x(\tau) = \int_{-\infty}^{\infty} S_x(\omega).e^{-i\omega\tau}.d\omega$ (17)

When τ is set to 0 in the equation, the fundamental property of $S_x(\omega)$ is revealed. $S_x(\omega)$ is then referred to as the spectral density of the *x* process, which is a function of angular frequency:

$$R_x(\tau = 0) = \int_{-\infty}^{\infty} S_x(\omega).d\omega \qquad (18)$$

3. Experimental model

Our investigation focuses on analyzing the vibration signal of the Saigon Bridge in Ho Chi Minh City to assess the cumulative damage that occurs over time. The bridge connects the city's urban and suburban areas and is comprised of 32 spans, with three middle spans (the 17th, 18th, and 19th) made of structural steel bearing beams, and the rest made of reinforced concrete, each measuring 24.7 m in length and 24 m in width. The Saigon Bridge plays a critical role as a major transportation link between Ho Chi Minh City and the southern provinces. To identify any changes in damage to the bridge, we conducted various measurements on its vibration signal. In this study, we present our findings on the Saigon Bridge.

Figure 1. A glimpse of the Saigon Bridge.

Figure 2. The project utilized an accelerometer sensor.

Table 1. Parameters of the vibrating accelerometer.

Sensor: Vibration accelerometer sensor (SENSR)	Process#: 224
Instrument: SENSR GP 1P	General accuracy: ± (50 mg ± 5% of reading)
Serial#: 1999	Measurement locations: midpoint of each span
Manufacturer: Reference LLC/sensor	Sampling speed: 100 Hz
Number of samples: 2000 samples per file and 30 files at each position	
External stimulation: traffic loads from 2011 to 2015 with six measurement campaigns (Table 1)	

Table 2. Measurement campaigns' time.

Campaign	1	2	3	4	5	6
Time	11/2011	02/2012	05/2012	08/2012	07/2013	10/2015

Tables 1 and 2 indicate that the duration between consecutive campaigns was about three months for the first four measurements. However, there was an interval of 11 months between the fourth and fifth campaigns and a duration of 27 months between the fifth and sixth campaigns.

Figure 3. Measuring at the midpoint of the 19th span.

4. Conclusions of the power spectrum density of real vibration signals

The signals under the moving loads on Saigon Bridge, which were continuously measured, are depicted in figure 4. The recorded vibrations exhibit diverse values of amplitude and duration, which can be attributed to the presence of multiple loads moving at different velocities simultaneously.

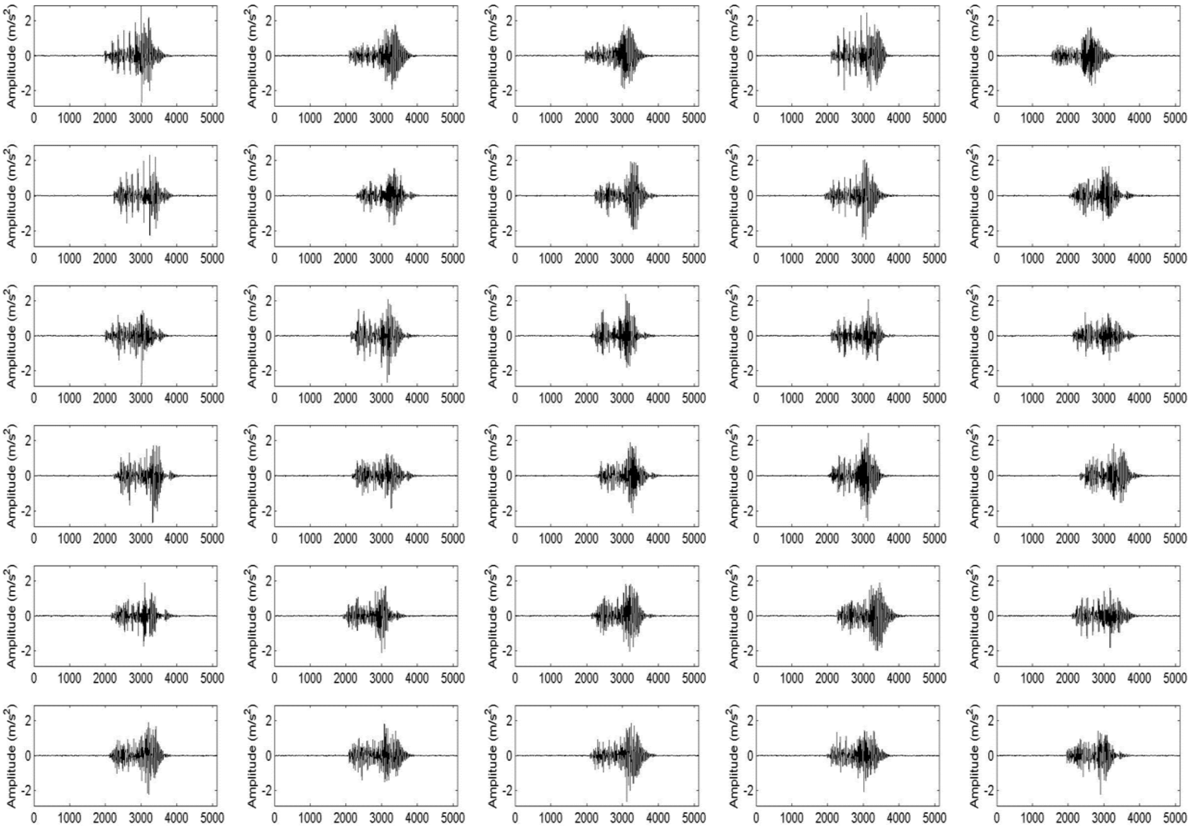


Figure 4. The acceleration vibration signal graphs of a single span on Saigon Bridge.

The power spectrum density of the actual vibration signals collected from the bridge datasets using Fourier analysis theory is shown in figure 5.

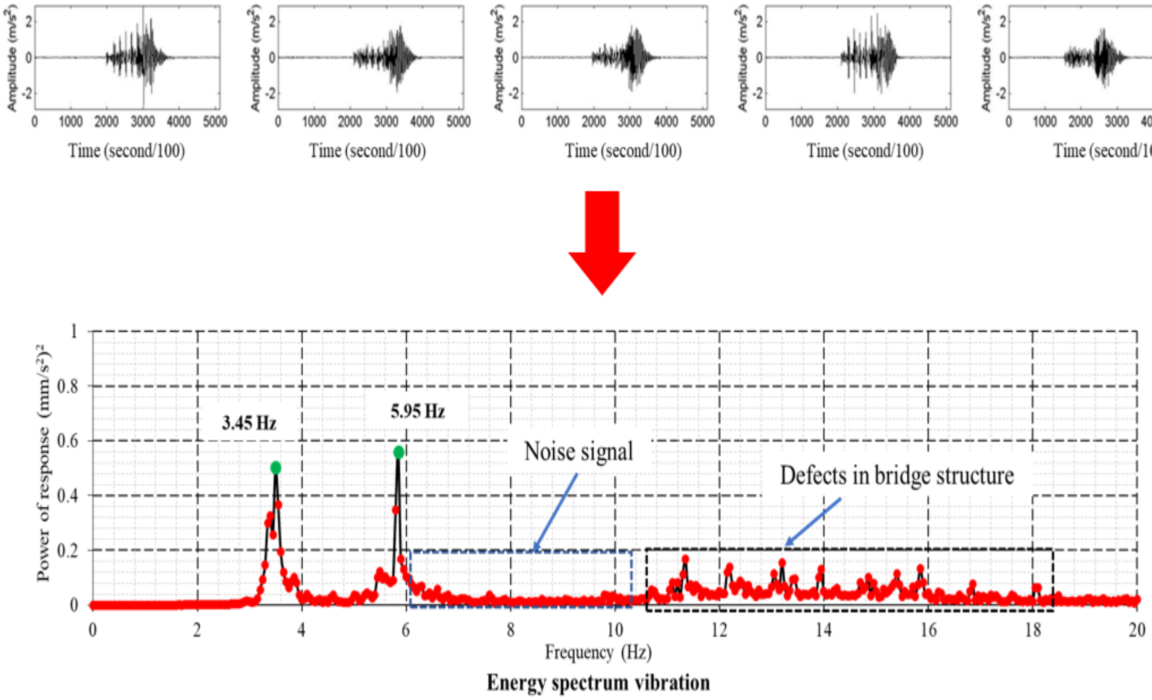


Figure 5. Graphs depicting the acceleration vibration signals observed at a single span of Saigon Bridge.

By applying Fourier analysis to the data represented in figure 4, the real vibration signal can be transformed into the power spectrum density shown in figure 6.

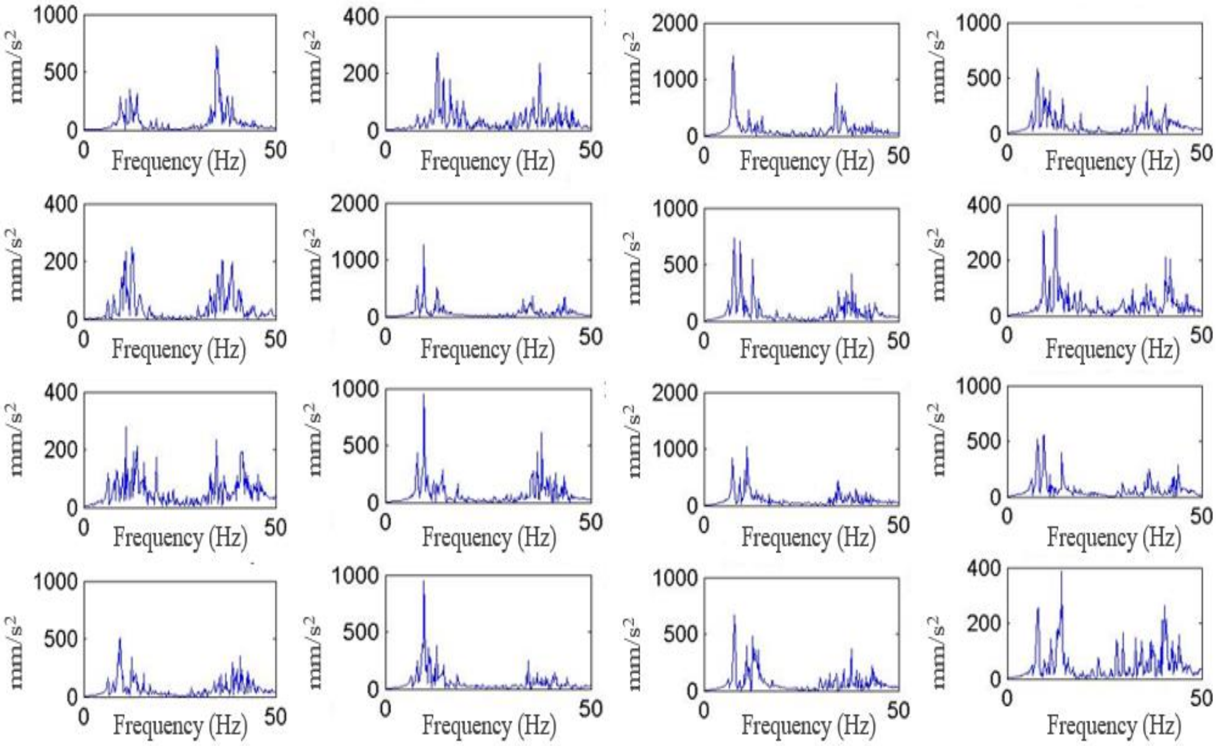
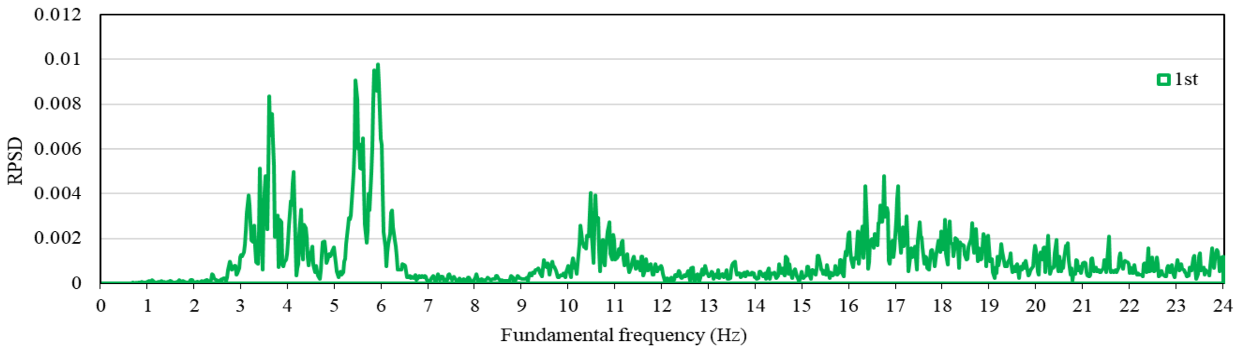


Figure 6. The power spectrum density of the vibration signals observed at a single span of the Saigon Bridge.

Based on the findings presented in Fig. 6, it is clear that the power spectrum density (PSD) of Saigon Bridge can be significantly influenced by variations in circulation during the measurement period. The manuscript provides a comprehensive analysis of the PSD characteristics of Saigon Bridge over a period of ten years, which yields the following observations:

- Among the total PSDs surveyed, which accounts for 20%, or 6 PSDs, all harmonics are present within a single resonance region spanning from 3 Hz to 6 Hz. This suggests that the power spectral densities (PSDs) can effectively represent the vibrational free state of the bridge by identifying a fundamental frequency that corresponds to the harmonic frequency with the highest amplitude. However, there are discrepancies in the frequency values of individual PSD graphs, which may arise from measurement or data processing errors or interference signals.
- The second observation made from the survey PSDs is that approximately 37% of them (10 PSDs) exhibit harmonics concentrated in two resonance regions. The first region spans from 3 Hz to 6 Hz, which is consistent with the fundamental frequency of the vibration freedom on the span observed in about 20% of the PSDs. The second region spans from 10 Hz to 12 Hz, indicating that frequencies within this range also have an impact on the vibration behavior.
- Last but not least, almost 43% of the total survey PSDs (14 PSDs) demonstrate harmonics that extend from 2 Hz to 24 Hz. Last but not least, almost 43% of the total survey PSDs (14 PSDs) demonstrate harmonics that extend from 2 Hz to 24 Hz.

Overall, the study provides insights into the dynamic behavior of the Saigon Bridge and emphasizes the importance of understanding resonance regions for effective bridge monitoring and maintenance.


RPSD
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Fundamental frequency (Hz)
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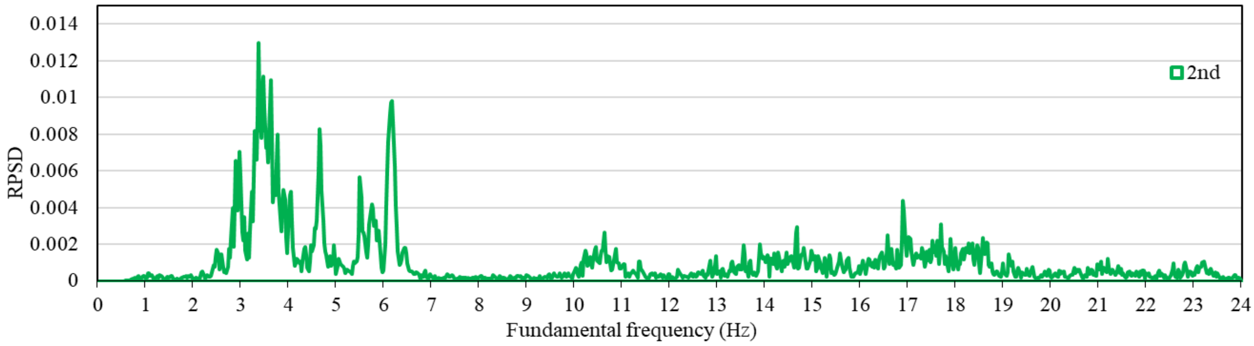
(a)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Fundamental frequency (Hz)
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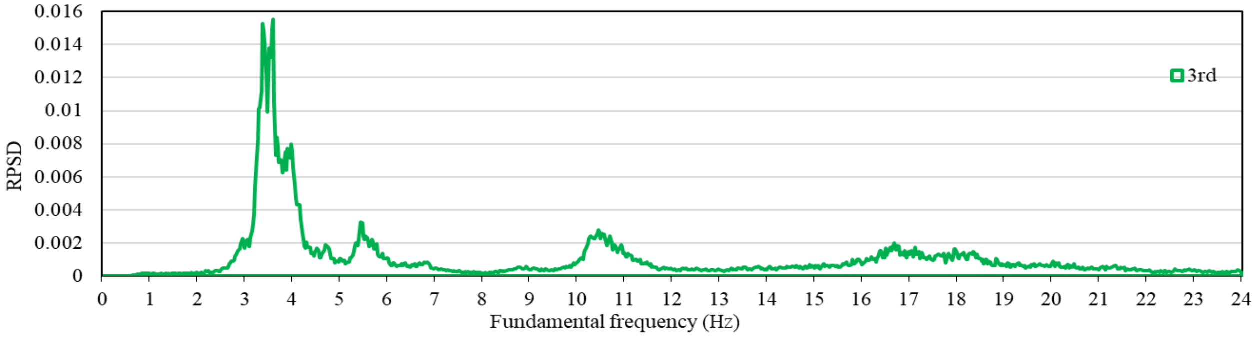
(b)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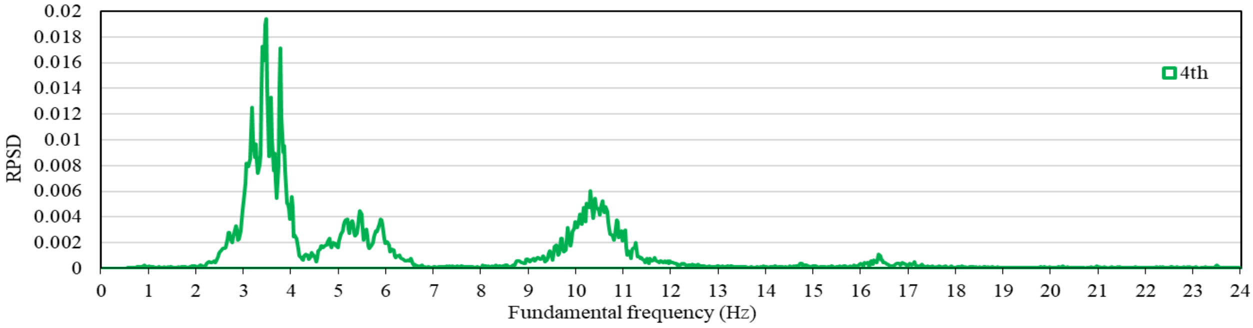
(c)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Fundamental frequency (Hz)
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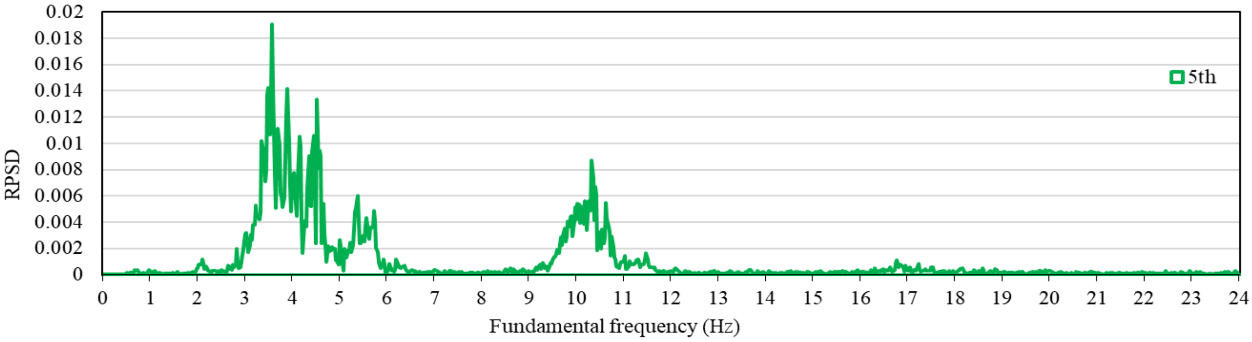
(d)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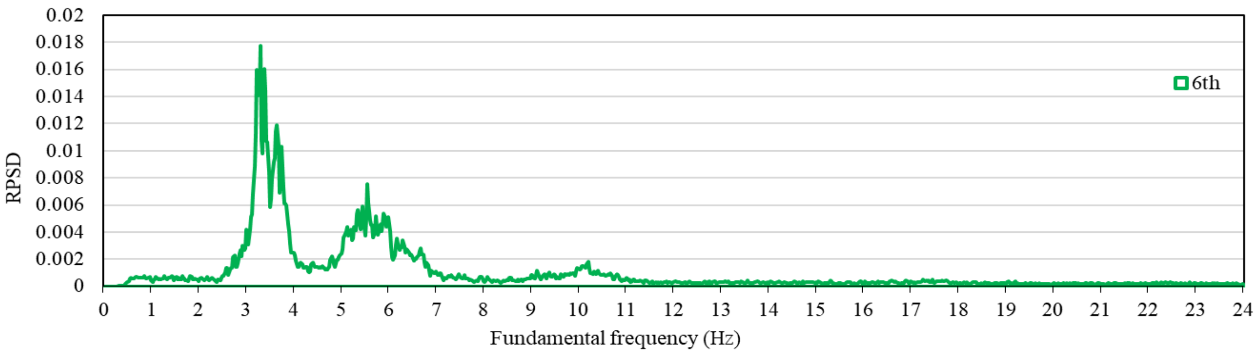
Fundamental frequency (Hz)
🟩5th

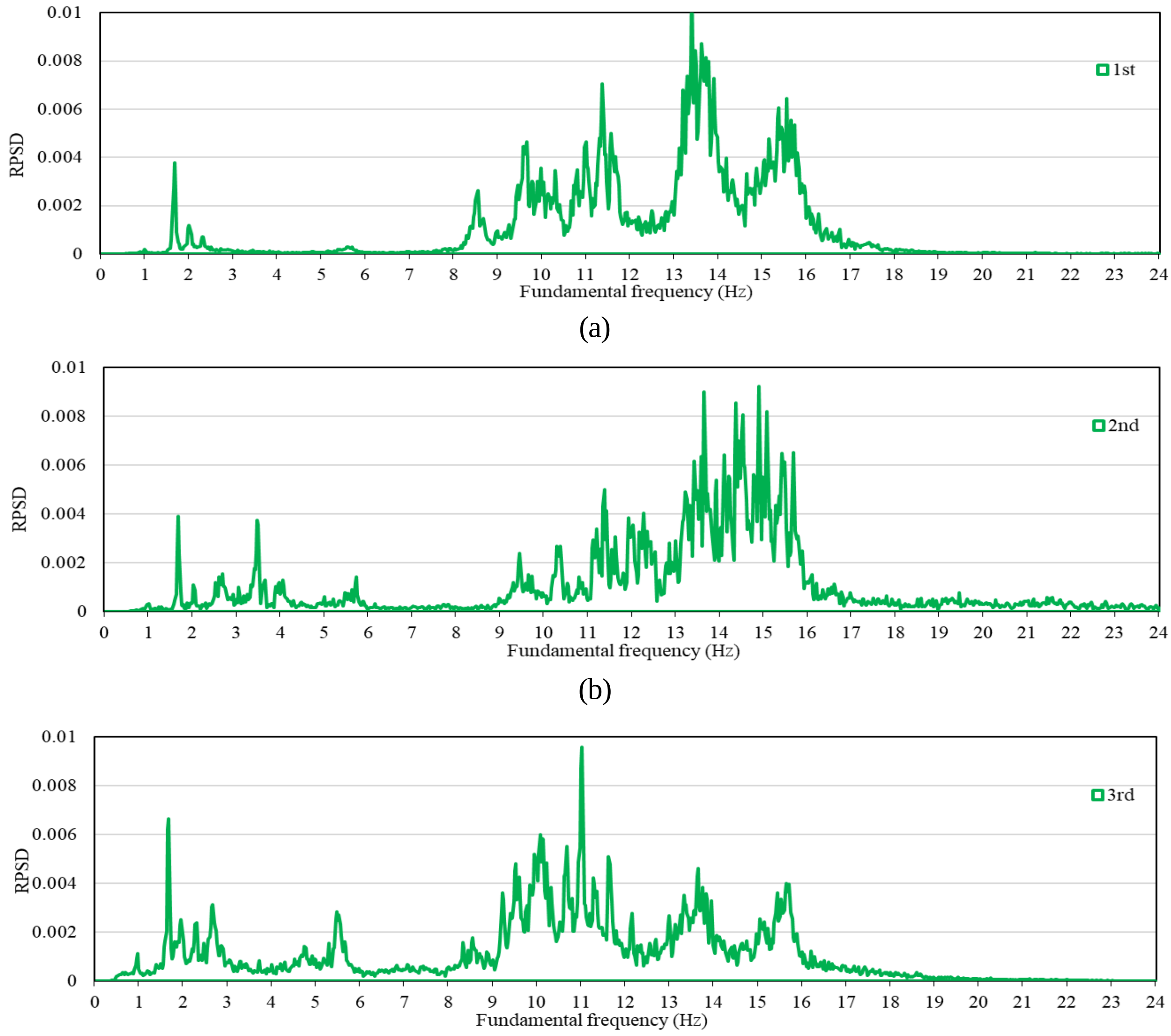
(e)



(f)

Figure 7. The PSD of the prestressed concrete material with measuring and analyzing in six different instances to investigate its dynamic characteristics.

A comprehensive analysis of the power spectrum density of the prestressed concrete material span was conducted in six different instances using a commonly used technique that decomposes a signal into its frequency components. The analysis identified three resonance regions with significant amplitude: the first ranging from 2.0 Hz to 6.0 Hz, the second ranging from 9.0 Hz to 12 Hz, and the third ranging from 14.5 Hz to 24.5 Hz. By providing this information, potential issues due to external factors can be identified, which in turn provides a better understanding of the structure's behavior.



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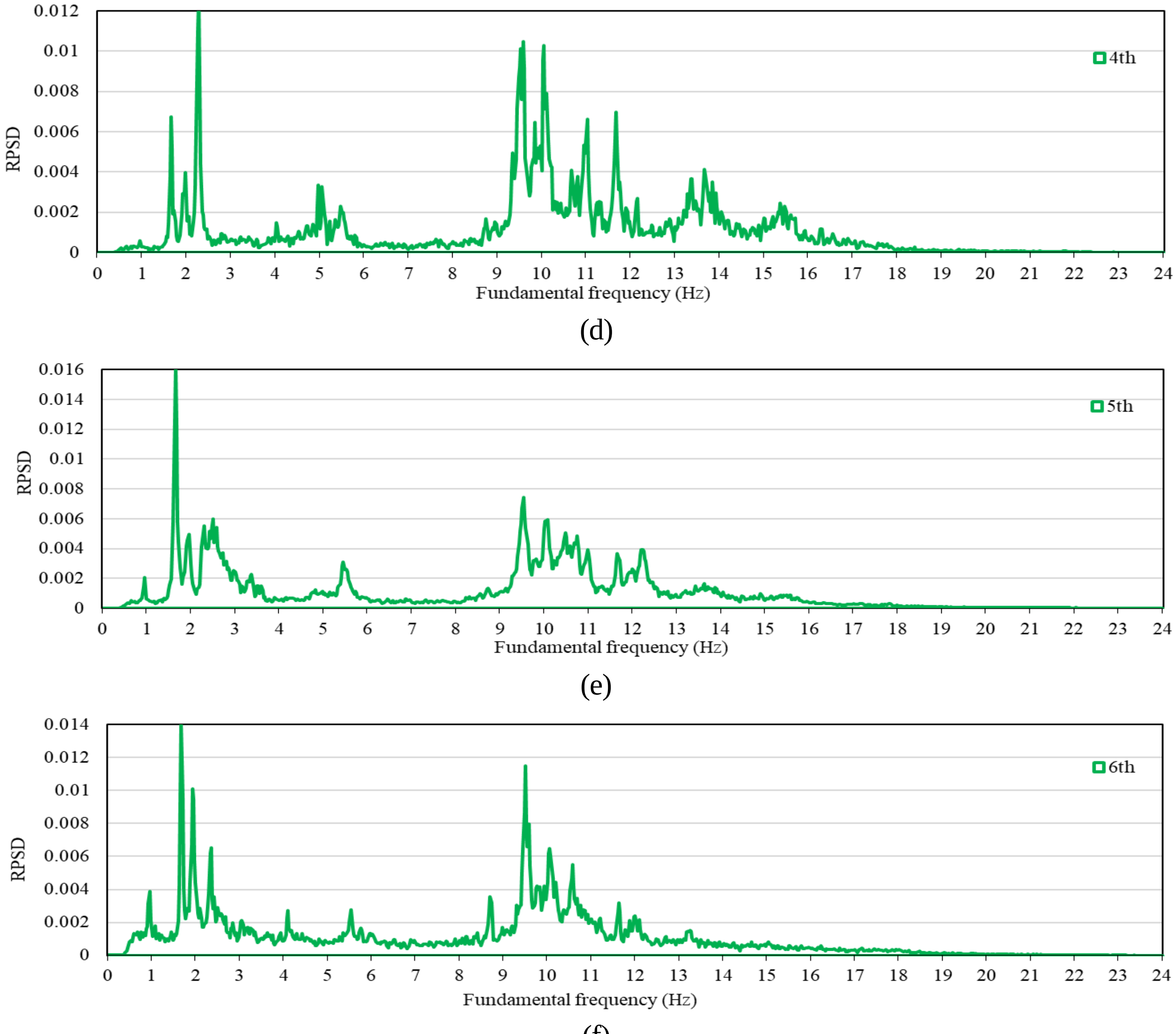

(d)

(e)

(f)

Figure 8. The dynamic characteristics of the structural steel material span by measuring its PSD Through six different instances.

Through the PSD measurements taken on Saigon Bridge, the bridge's vibrational freedom state was found to be characterized by multiple resonance regions with different frequencies and amplitudes. Although the lowest resonance region has a lower frequency value, it was observed to be the most stable over time and appeared in all PSD measurements. The highest amplitude was observed in the first resonance region, while the third resonance region's amplitude and area decreased over time and disappeared after multiple measurements.

Furthermore, the PSD measurements revealed that the span's vibrational freedom state changed over time, with the PSD sharing varying with operating time. The study highlighted that factors such as the material used, measurement error, and data processing could influence the vibrational freedom state of the bridge. As such, regular PSD measurements can provide valuable insights into the state of a bridge's vibrational freedom and help inform maintenance and repair decisions.

5. Conclusions

The study conducted on Saigon Bridge produced the following outcomes:

(i) In Vietnam, bearing capacity testing is a commonly used method for assessing the operational condition of bridges, providing a structural analysis. By analyzing changes in the power spectrum density's shape using random vibration signals, researchers assessed the bearing capacity of Saigon Bridge. The information gathered through this method can be highly useful for making informed decisions regarding maintenance and repair.

(ii) Real vibration signals derived from a randomized traffic load model were used by the researchers to determine the power spectral density. The study found a correlation between the appearance of harmonics and high-frequency regions and the gradual reduction in stiffness of the bridge's spans over time. This slow deterioration makes it difficult to monitor the bridge's condition accurately, creating a challenge.

References

[1] J. R. Maguire; "Assessing the Dynamic Properties of Existing Bridge Structures by Hammer Testing"; Bridge Management-Inspection, Maintenance, Assessment and Repair; Boston, USA:Springer; 1990, pp.596-605.

[2] J. K. Vandiver, "Dimensionless parameters important to the prediction of vortex-induced vibration of long, flexible cylinders in ocean currents". Journal of Fluids and Structures. 1993; 7(5):423-455. DOI:10.1006/jfls.1993.1028.

[3] J. K. Vandiver, "Detection of Structural Failure on Fixed Platforms by Measurement of Dynamic Response". Journal of Petroleum Technology. 1977, 29(3):305-310. DOI:10.2118/5679-PA.

[4] R. M. Kenley and C. J. Dodds; "West Sole WE Platform: Detection of Damage by Structural Response Measurements"; in Proc. of 12th Annual Offshore Technology Conference; 5-8 May 1980; Houston, Texas, USA; 1980 pp. 111-118.

[5] J. J. Tracy and G. C. Pardoen, "Effect of Delamination on the Natural Frequencies of Composite Laminates". Journal of Composite Laminates. 1989; 23(12):1200-1215. DOI:10.1177/002199838902301201.

[6] X. C. Man, L. M. McClure, Z. Wang, R. D. Finch, P. Y. Robin, B. H. Jansen; "Slot Depth Resolution in Vibration Signature Monitoring of Beams Using Frequency Shift". Journal of the Acoustical Society of America. 1994; 95(4):2029-2037. DOI:10.1121/1.408666.

[7] F. K. Choy, R. Liang and P. Xu; "Faull Identification of Beams on Elastic Foundation". Computers and Geolechnics. 1995; 17(2):157-176. DOI:10.1016/0266-352X(95)93867-I.

[8] G. Hearn and R. B. Testa; "Modal Analysis for Damage Detection in Structure". Journal of Structural Engineering. 1991; 117(10):3042-3063. DOI:10.1061/(ASCE)0733-9445(1991)117:10(3042).

[9] D. R. Sanders, Y. I. Kim and R. N. Stubbs; "Non-destructive Evaluation of Damage in Composite Structures using Modal Parameters". Experimental Mechanics. 1992; 32: 240-251. DOI:10.1007/BF02319362.

[10] P. S. Skjærbæk, S. R. K. Nielsen, P. H. Kirkegaard, A. Ş. Çakmak; "Damage localization and quantification of earthquake excited RC-frames". Earthquake Engineering and Structural Dynamics. 1998; 27(9):903-916. DOI: 10.1002/(SICI)1096-9845(199809)27:9<903::AID-EQE757>3.0.CO;2-C.

[11] N. T. Khiem and T. V. Liem; "A simplified method for natural frequency analysis of a multiple cracked beam". Journal of Sound and Vibration. 2001; 245(4):737-751. DOI:10.1006/jsvi.2001.3585.

[12] J. K. Sinha, M. I. Friswell and S. Edwards; "Simplified models for the location of cracks in beam structures using measured vibration data". Journal of Sound and Vibration. 2002; 251(1):12-38. DOI:10.1006/jsvi.2001.3978.

[13] T. Q. Nguyen, T. D. Nguyen, L. Q. Tran, N. K. Ngo; "A New Insight to Vibration Characteristics of Spans under Random Moving Load: Case Study of 38 Bridges in Ho Chi Minh City, Vietnam". Shock and Vibration. 2020; 2020:1547568. DOI:10.1155/2020/1547568

[14] T. Q. Nguyen, T. A. Nguyen, T. T. Nguyen; "PSD characteristics for the random vibration signals used in bridge structural health monitoring in Vietnam based on a multi-sensor system". International Journal of Distributed Sensor Networks. 2022; 18(9). DOI:10.1177/15501329221125.