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Quantum optimization in autonomous underwater vehicle routing navigation

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ABSTRACT

Quantum optimization is currently developing strongly in many research fields. This method uses the principle of quantum mechanics to determine the process of quantum fluctuations in optimal calculations. In this study, we use the Quantum Annealing Algorithm (QAA) to optimize the journey of the Autonomous Underwater Vehicle (AUV) to find the shortest distance, consume the least energy, and even avoid obstacles while moving. The research results on simulations are compared with the traditional Dijkstra method to evaluate the effectiveness of the proposed method, thereby affirming the superiority of quantum optimization in the automatic control of AUV.

1. Introduction

Automatic control and applications are becoming more common as ocean technology advances, with autonomous underwater vehicles being a modern way that is growing frequently due to their benefits. Applications include ocean exploration [1], underwater monitoring [2], and marine environments [3], where optimization-oriented routing control is gaining popularity among academics. There are numerous difficulties and challenges in the process of controlling AUV routings, such as the limitation of wireless communication in the water environment, high energy consumption resulting in short independent operation time, and the random influence of the environment, which can efficiently lead to loss of control of the AUV. On the other hand, obstacle avoidance studies are of interest in robotics technology in industries because they increase the safety and sustainability of robot operations while preventing uncommon accidents that may arise during operation. With AUV controllers and energy limits, we always demand the fewest operations possible to optimize sustainability while achieving a specific target function [4], [5]. As a result, AUV control calculates the ideal distance, avoids

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impediments as soon as they are recognized, and conserves energy. Therefore, this issue presents a problem for scientists researching AUV routing control.

AUV routing control focuses on key points, which are control quality, accuracy, and sustainability in autonomous operation. Conventional AUVs are equipped with a limited-capacity electric battery, but autonomous underwater operations often take a long time due to environmental factors that prevent the AUV from ending its journey as planned, which can cause a decrease in power leading to loss of control or incorrect mission performance. Moreover, to avoid collisions, the AUV should stay away from obstacles such as ships, coral reefs, and underwater or offshore structures [6], [7]. This further enhances the importance of optimal control algorithms. If we limit the research problem here, without mentioning the quality of the controller, and consider that the AUV has a good controller, then determining the direction will determine whether the AUV's operation is optimal or not. Actually, classical routing methods, such as the Dijkstra algorithm [8], which effectively calculates the shortest path based on distance, have been studied and applied. Recently, the A* algorithm has been upgraded from the classical Dijkstra algorithm [9] to increase the calculation speed, thereby reducing the energy loss for controllers and actuators. New optimization algorithms, such as Genetic Algorithms (GAs) [10] and Particle Swarm Optimization (PSO) [11], have helped increase efficiency by allowing the path to be updated to adapt to changes in the environment. It can be seen that with the development of computational science and the speed of digital computers, new algorithms with superior features will replace old algorithms, which is the inevitable trend of the current era.

Quantum optimization algorithms (QOA) are widely used in many fields of automation, such as control in transportation, logistics, robotics, industry, mining, space exploration, ocean exploration, and the offshore sector. By taking advantage of quantum principles, the QOA aims to solve optimization problems with many constraints, especially those that require global optimization to achieve the value of the loss function. Eventually, when QOA is integrated with powerful, fast hardware such as quantum computers, the problem to be solved becomes enormous. A recent study on the application of maritime routing and risk mitigation in collision avoidance [12] has shown the effectiveness of QOA as well as the future direction of this method.

In this study, we first analyze the routing problem for the AUV to plan its optimal movement, in which the issues of environmental and weather influences and obstacle avoidance are considered in general. Next, there are currently many QOAs being researched and developed, among which the Quantum Annealing Algorithm (QAA) is selected for testing and proving its effectiveness by simulation when compared with the traditional Dijkstra algorithm. The goal is to achieve the optimal QAA functionality in which, as mentioned above, the AUV's travel route is the shortest, with the least energy loss. At the same time, when obstacles are randomly distributed on the map, the AUV calculates the path to avoid colliding with them.

2. AUV routing navigation framework

Considering the 3D space corresponding to the operation of AUVs at sea, the routing operation is considered by calculating the destinations of the AUV in the critical space, the number of destinations being the locations that the AUV will be forced to pass through once. The problem can be applied to other vehicles, such as ships when moving on the sea surface, at which time we will choose a 2D coordinate system.

2.1. Components of the AUV routing navigation model

Suppose we have a set of stops set as $\boldsymbol{W} = \{\boldsymbol{w_1}, \boldsymbol{w_2}, \dots, \boldsymbol{w_N}\}$, corresponding to N destinations; this shows that the AUV will have a journey passing through these *N* points and will have *N-1* travel distances. The distance between the destinations is determined through the distance weight matrix $\boldsymbol{D(i,j) = d_{ij}}$, which represents the distance from point I to some point j. Normally, to calculate the distance from point to point in 3D space, many authors [13], [14], [15] use the Euclid formula as follows:

$$d_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2 + (z_i - z_j)^2} \quad (1)$$

Where $(\boldsymbol{x_i}, \boldsymbol{y_i}, \boldsymbol{z_i})$ is the position in the AUV's moving space on the sea. By computing and optimizing the sum of *N-1* distances that connect *N* destinations, we will determine the best route to maneuver the AUV.

2.2. Problem to be solved

Regarding the speed of the AUV moving steadily, the travel time is the least when the distance is the shortest, which means the energy loss of the AUV is the smallest. Then, we will achieve the optimal condition with the constraints mentioned above, such as the AUV's limited moving map, environmental conditions, ocean currents, or the maximum impacts on the AUV so that the AUV can maintain its automatic direction. In this study, it is assumed that the optimal AUV route is $\boldsymbol{P^*} = \{\boldsymbol{p_1}, \boldsymbol{p_2}, \dots, \boldsymbol{p_N}\}$, which is defined as the route sequence that minimizes the total distance $\boldsymbol{C(P)}$ as follows:

$$C(P) = \sum_{i=1}^{N-1} d_{p_i p_{i+1}} \quad (2)$$

3. Quantum Annealing Route Path Optimal Algorithm

3.1. Quantum Annealing formulation

The AUV route optimization problem, according to equation (2), can be calculated with the objective function being the minimum total path length of the AUV (3) as follows:

$$\min \sum_{i=1}^{N-1} w_{i,i+1} \quad (3)$$

Where $\boldsymbol{w_{i,i+1}}$ is the distance between two consecutive waypoints on the path, and $\boldsymbol{N}$ is the total number of stops. Typically, we use the Hamiltonian [16] of the optimal path-finding problem, which can be written as follows:

$$H_P = \sum_{i=1}^{N-1} w_{i,i+1} x_i x_{i+1} \quad (4)$$

Where $\boldsymbol{x_i}$ is a binary variable representing the presence of a waypoint *i* on the path. In practice, the search for the optimal solution of the Hamiltonian (4) follows the Schrödinger equation [16]:

$$\frac{d}{dt}\Psi(t) = -iH(t)\Psi(t) \quad (5)$$

Where $\boldsymbol{\Psi(t)}$ is the quantum state, and $\boldsymbol{H(t)}$ is a time-varying Hamiltonian. Next, the optimization process proceeds by changing from $\boldsymbol{H_0}$ to $\boldsymbol{H_P}$ in time *T*:

$$H(t) = \left(1 - \frac{t}{T}\right) H_0 + \frac{t}{T} H_P \quad (6)$$

Finally, the process will reach the lowest energy state, corresponding to the optimal path-path when $\boldsymbol{t} \rightarrow \boldsymbol{T}$.

3.2. Quantum Annealing Algorithm

Using equations (3), (4), (5), and (6), aiming to reach the optimal goal with the lowest energy, we proposed Algorithm 1 with the output of the total moving distance of the AUV corresponding to the lowest energy loss in this study. Then, start by setting the distance matrix between the waypoints, designing the map size, and setting a first point to move.

Algorithm 1. Quantum Annealing Algorithm

```
Input: Set t ← 0, num_waypoints ← 100, num_obstacles ← 10; map_size = 10.000; energy_limit ← 5000
Output: total distance; energy loss; cost function
    Initialize Creating a distance matrix $w_{ij}$ between waypoints
    Initialize Initializing a map size, cooling rate $\alpha$
    Initialize Initializing a random initial path
    Initialize Current_path & [1, 2, ..., num_waypoints]
            best_path & current_path;
            best_dist * Comp_Total_Dist(dist_matrix, best_path)
    FUNCTION QuantumAnnealing(dist_matrix, num_waypoints)
    Initialize T0 ← 100, α ← 0.01, M ← 5000, T ← T0
    for iter from 1 to M do
        Select random indices (a, b) Where a ≠ b
        swap_idx = randperm(num_waypoints-2, 2) + 1
        swap_idx ← RandomSwapIndices()
        new_path ← SwapElements(best_path, swap_idx)
        new_dist ← Comp_Total_Dist(dist_matrix, new_path).
                          $\sum_{i=1}^{N-1} w_{i,i+1}$                    (7)
        new_path ← SwapElements(best_path, a, b)
         if new_dist < best_dist then
           best_path ← new_path
           best_dist ← new_dist
         else
         Acceptance_Prob ← exp((best_dist - new_dist) / T);
        (If the new path is longer, use the Metropolis-Hastings probability to decide whether to accept it or not)
            if Random (0,1) < Acceptance_Prob then
                best_path ← new_path
                best_dist ← new_dist
            end if
```

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            end if
            Lower the emulator's temperature.
            $T \leftarrow T0 * \exp(-\alpha*iter)$
        end for
        RETURN best_path, best_dist
    END FUNCTION
End
```

4. Simulation and comparison

4.1. Simulation setup

The AUV route optimization problem in an idealized environment is represented as a graph problem where each waypoint is defined as a vertex, with the edges weighted according to the distances between each of the waypoints. Moreover, we aim to find the path with the least cumulative travel distance, considering important considerations of avoiding obstacles, effects of hydrodynamic flow, and energy restrictions. For comparison, we apply two algorithms in this simulation: the Dijkstra algorithm [17], which utilizes an efficient weighted graph shortest path computation, and the proposed QAA in Algorithm 1.

$$C_i = 0.1 \times \mathcal{N}(0,1) \tag{8}$$

Where $\boldsymbol{C_i}$ is the ocean current intensity at waypoint $\boldsymbol{i}$, and also $\boldsymbol{\mathcal{N}(0,1)}$ is a random variable. The simplest wave model is the following harmonic wave equation:

$$w_i = A\sin(2\pi f t + \phi) \tag{9}$$

Where $\boldsymbol{w_i}$ is the displacement caused by waves at waypoint $\boldsymbol{i}$; $\boldsymbol{A}$ is the wave amplitude (m); $\boldsymbol{f}$ is the wave frequency (**Hz**); $\boldsymbol{\phi}$ is a random phase shift. Moreover, the AUV setup included a velocity range of 0.5 to 1 m/s and a total weight of 50 kg. Assuming AUV operates in good weather and calm sea conditions, we choose a frequency around 0.1 Hz with a wave amplitude similar to 0.5m in simulation.

4.2. Results

Matlab simulations conducted to evaluate the effectiveness of the Dijkstra method against the proposed QAA. In this section, we choose a simpler AUV motion problem, considering only AUV motion on the sea surface (0XY coordinate system).

The simulation results by the distance traveled of the AUV after running the QAA optimization algorithm and Dijkstra algorithm on Matlab and the experiments include three scenarios involving 10 waypoints (Figure 1), 15 waypoints (Figure 2), and 20 waypoints (Figure 3). The calculation of the total distance, energy consumption, and total cost are based on [18] and are compared in Tables 1, 2, and 3, respectively. From the above data and results, we can see that QAA achieves better results than Dijkstra. In conclusion, as the number of waypoints increases, QAA becomes more effective.

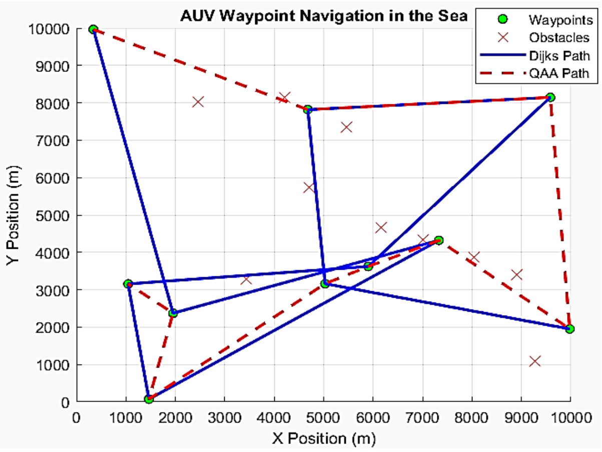

Figure 1. The AUV moving on the path includes 10 waypoints and avoids 10 obstacles.

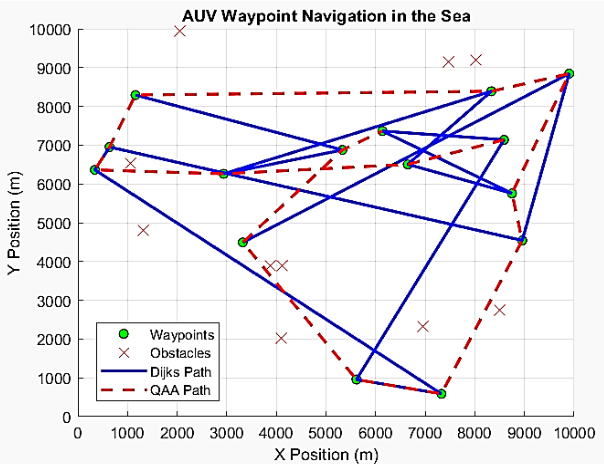

Figure 2. The AUV moving on the path includes 15 waypoints and avoids 10 obstacles.

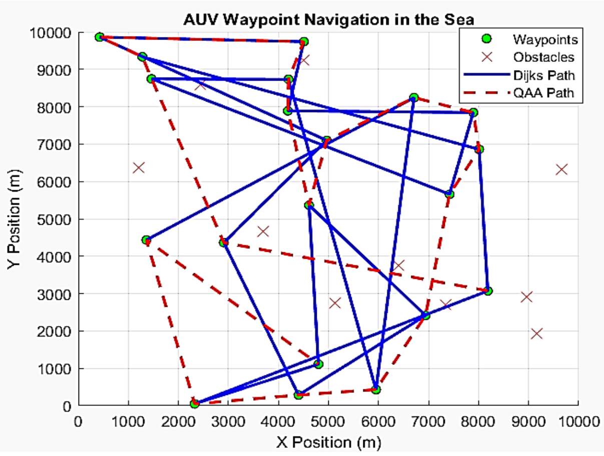

Figure 3. The AUV moving on the path includes 20 waypoints and avoids 10 obstacles.

Table 1. Performance metrics of Algorithms - case 10 waypoints.

Algorithms	Total Distance (m)	Energy Loss (kJ)	Cost Function
Dijkstra	42.208	3.054,261	1.569.339
QAA	28.479	2.060,825	1.058.892

Table 2. Performance metrics of Algorithms - case 15 waypoints.

Algorithms	Total Distance (m)	Energy Loss (kJ)	Cost Function
Dijkstra	62.243	4.262,901	2.193.693
QAA	38.090	2.608,749	1.342.465

Table 3. Performance metrics of Algorithms - case 20 waypoints.

Algorithms	Total Distance (m)	Energy Loss (kJ)	Cost Function
Dijkstra	88.885	5.439,166	2.808.469
QAA	46.053	2.818,113	1.455.109

4.3. Discussion

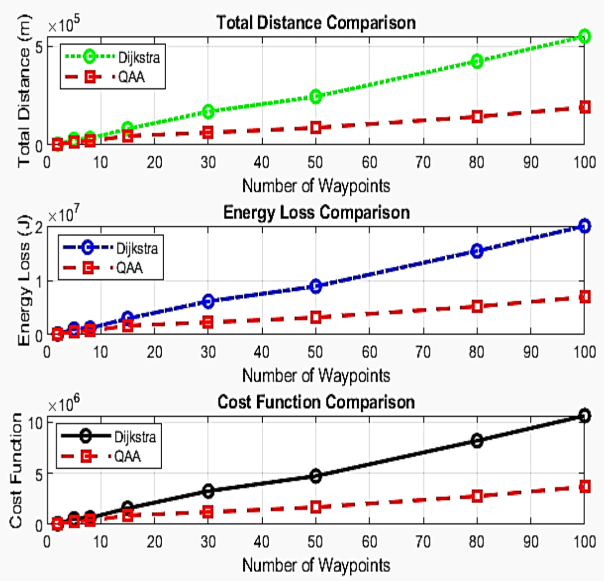

Figure 4. The AUV moving on the path includes 20 waypoints and avoids 10 obstacles.

Continuing to test the cases of 30 waypoints, 50 waypoints, 80 waypoints, and finally 100 waypoints, respectively, the comparison simulation results between the two algorithms are shown in Figure 4. Therefore, we can see that the number of waypoints is large, Dijkstra cannot ensure optimal performance and distance, while QAA proves effective. It can be seen that the quantum optimization method is very suitable for objects with complex and frequently changing trajectories. However, the QAA algorithm will be developed for stable or adaptive control problems, predicting that it can achieve very good results compared to GA or PSO.

5. Conclusion

In this study, we first analyze the routing problem for the AUV to plan its optimal movement, in which the issues of environmental and weather influences and obstacle avoidance are generally considered. Next, there are currently many QOAs being researched and developed, among which the QAA is selected for testing and proving its effectiveness by simulation when compared with the traditional Dijkstra algorithm. The goal is to achieve the optimal QAA functionality in which, as mentioned above, the AUV's travel route is the shortest, with the least energy loss. At the same time, when obstacles are randomly distributed on the map, the AUV calculates the path to avoid colliding with them. On the other hand, the proposed QAA can be completed and applied in practice when obstacles are mobile and uncertain, and it is necessary to apply additional real-time collision avoidance algorithms.

Contributions of authors in this article

Dang Xuan Kien: Methodology, Data management, Formal analysis, Investigation, Validation, Visualization, Feedback on peer review, Revising – original manuscript. **Ngoc-Ha Vu**: Data compilation, Simulation, Data analysis, Investigation, Verification, Writing – original manuscript.

Conflict of Interest and Copyright Statement

The author declares that there are no potential conflicts of interest arising from this study, and that the article has not been previously published.

Data available upon request

Data will not be provided upon request.

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